NAG C Library Function Document

nag_ztbsv (f16skc)

1 Purpose

nag_ztbsv (f16skc) solves a system of equations given as a complex triangular band matrix.

2 Specification

```
#include <nag.h>
#include <nagf16.h>
```

```
void nag_ztbsv (Nag_OrderType order, Nag_UploType uplo, Nag_TransType trans,
     Nag_DiagType diag, Integer n, Integer k, Complex alpha, const Complex ab[],
     Integer pdab, Complex x[], Integer incx, NagError *fail)
```

3 Description

nag_ztbsv (f16skc) performs one of the matrix-vector operations

$$x \leftarrow \alpha A^{-1}x, \quad x \leftarrow \alpha A^{-\mathrm{T}}x \quad \text{or} \quad x \leftarrow \alpha A^{-\mathrm{H}}x,$$

where A is an n by n complex triangular band matrix with k subdiagonals or superdiagonals, x is an n element complex vector and α is a complex scalar. $A^{-\mathrm{T}}$ denotes $\left(A^{\mathrm{T}}\right)^{-1}$ or equivalently $\left(A^{-1}\right)^{\mathrm{T}}$; $A^{-\mathrm{H}}$ denotes $\left(A^{\mathrm{H}}\right)^{-1}$ or equivalently $\left(A^{-1}\right)^{H}$.

No test for singularity or near-singularity of A is included in this function. Such tests must be performed before calling this function.

4 References

The BLAS Technical Forum Standard (2001) *www.netlib.org/blas/blast-forum*

5 Arguments

1: **order** – Nag_OrderType *Input*

On entry: the **order** argument specifies the two-dimensional storage scheme being used, i.e., row-major ordering or column-major ordering. C language defined storage is specified by **order** = **Nag_RowMajor**. See Section 2.2.1.4 of the Essential Introduction for a more detailed explanation of the use of this argument.

Constraint: **order** = **Nag_RowMajor** or **Nag_ColMajor**.

2: **uplo** – Nag_UploType *Input*

On entry: specifies whether A is upper or lower triangular.

uplo = **Nag_Upper**

A is upper triangular.

uplo = **Nag_Lower**

A is lower triangular.

Constraint: **uplo** = **Nag_Upper** or **Nag_Lower**.

3: **trans** – Nag_TransType *Input*

On entry: specifies the operation to be performed.

trans = **Nag_NoTrans**

$x \leftarrow \alpha A^{-1}x$.

trans = **Nag_Trans**

$x \leftarrow \alpha A^{-\mathrm{T}}x$.

trans = **Nag_ConjTrans**

$x \leftarrow \alpha A^{-\mathrm{H}}x$.

Constraint: **trans** = **Nag_NoTrans**, **Nag_Trans** or **Nag_ConjTrans**.

4: **diag** – Nag_DiagType *Input*

On entry: specifies whether A has non-unit or unit diagonal elements.

diag = **Nag_NonUnitDiag**

The diagonal elements are stored explicitly.

diag = **Nag_UnitDiag**

The diagonal elements are assumed to be 1 and are not referenced.

Constraint: **diag** = **Nag_NonUnitDiag** or **Nag_UnitDiag**.

5: **n** – Integer *Input*

On entry: n, the order of the matrix A.

Constraint: $\mathbf{n} \geq 0$.

6: **k** – Integer *Input*

On entry: k, the number of subdiagonals or superdiagonals of the matrix A.

Constraint: $\mathbf{k} \geq 0$.

7: **alpha** – Complex *Input*

On entry: the scalar α.

8: **ab**[dim] – const Complex *Input*

Note: the dimension, *dim*, of the array **ab** must be at least $\max(1, \mathbf{pdab} \times \mathbf{n})$.

On entry: the n by n triangular band matrix A. This is stored as a notional two-dimensional array with row elements or column elements stored contiguously. The storage of elements a_{ij} depends on the **order** and **uplo** arguments as follows:

if **order** = **Nag_ColMajor** and **uplo** = **Nag_Upper**,
a_{ij} is stored in **ab**$[k+i-j+(j-1)\times$ **pdab**$]$,
for $j = 1,\ldots,n$ and $i = \max(1, j-k),\ldots,j$;
if **order** = **Nag_ColMajor** and **uplo** = **Nag_Lower**,
a_{ij} is stored in **ab**$[i-j+(j-1)\times$ **pdab**$]$,
for $j = 1,\ldots,n$ and $i = j,\ldots,\min(n, j+k)$;
if **order** = **Nag_RowMajor** and **uplo** = **Nag_Upper**,
a_{ij} is stored in **ab**$[j-i+(i-1)\times$ **pdab**$]$,
for $i = 1,\ldots,n$ and $j = i,\ldots,\min(n, i+k)$;
if **order** = **Nag_RowMajor** and **uplo** = **Nag_Lower**,
a_{ij} is stored in **ab**$[k+j-i+(i-1)\times$ **pdab**$]$,
for $i = 1,\ldots,n$ and $j = \max(1, i-k),\ldots,i$.

9: **pdab** – Integer *Input*

On entry: the stride separating row or column elements (depending on the value of **order**) of the matrix A in the array **ab**.

Constraint: **pdab** $\geq$ **k** $+ 1$.

10: **x**[dim] – Complex *Input/Output*

Note: the dimension, *dim*, of the array **x** must be at least $\max(1, 1 + (\mathbf{n} - 1)|\mathbf{incx}|)$.

On entry: the vector x.

On exit: the solution vector x.

11: **incx** – Integer *Input*

On entry: the increment in the subscripts of **x** between successive elements of x.

Constraint: **incx** $\neq 0$.

12: **fail** – NagError * *Input/Output*

The NAG error argument (see Section 2.6 of the Essential Introduction).

6 Error Indicators and Warnings

NE_BAD_PARAM

On entry, argument ⟨*value*⟩ had an illegal value.

NE_INT

On entry, **incx** $=$ ⟨*value*⟩.
Constraint: **incx** $\neq 0$.

On entry, **k** $=$ ⟨*value*⟩.
Constraint: **k** ≥ 0.

On entry, **n** $=$ ⟨*value*⟩.
Constraint: **n** ≥ 0.

NE_INT_2

On entry, **pdab** $=$ ⟨*value*⟩, **k** $=$ ⟨*value*⟩.
Constraint: **pdab** $\geq$ **k** $+ 1$.

7 Accuracy

The BLAS standard requires accurate implementations which avoid unnecessary over/underflow (see Section 2.7 of The BLAS Technical Forum Standard (2001)).

8 Further Comments

None.

9 Example

Solves complex triangular banded system of linear equations, $Ax = y$, where A is a complex triangular 4 by 4 matrix, with 2 subdiagonals, given by

$$A = \begin{pmatrix} -1.94 + 4.43i & & & \\ -3.39 + 3.44i & 4.12 - 4.27i & & \\ 1.62 + 3.68i & -1.84 + 5.53i & 0.43 - 2.66i & \\ & -2.77 - 1.93i & 1.74 - 0.04i & 0.44 + 0.10i \end{pmatrix}$$

and

$$y = \begin{pmatrix} -8.86 - 3.88i \\ -15.57 - 23.41i \\ -7.63 + 22.78i \\ -14.74 - 2.40i \end{pmatrix}.$$

9.1 Program Text

```
/* nag_ztbsv (f16skc) Example Program.
 *
 * Copyright 2005 Numerical Algorithms Group.
 *
 * Mark 8, 2005.
 */

#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nagf16.h>
#include <nagx04.h>

int main(void)
{

  /* Scalars */
  Complex alpha;
  Integer exit_status, i, incx, j, kd, n, pdab, xlen;

  /* Arrays */
  Complex *ab=0, *x=0;
  char nag_enum_arg[40];

  /* Nag Types */
  NagError fail;
  Nag_OrderType order;
  Nag_TransType trans;
  Nag_UploType uplo;
  Nag_DiagType diag;

#ifdef NAG_COLUMN_MAJOR
#define AB_UPPER(I,J) ab[(J-1)*pdab + kd + I - J]
#define AB_LOWER(I,J) ab[(J-1)*pdab + I - J]
#define B(I,J) b[(J-1)*pdb + I - 1]
  order = Nag_ColMajor;
#else
#define AB_UPPER(I,J) ab[(I-1)*pdab + J - I]
#define AB_LOWER(I,J) ab[(I-1)*pdab + kd + J - I]
#define B(I,J) b[(I-1)*pdb + J - 1]
  order = Nag_RowMajor;
#endif

  exit_status = 0;
  INIT_FAIL(fail);

  Vprintf( "nag_ztbsv (f16skc) Example Program Results\n\n");

  /* Skip heading in data file */
  Vscanf("%*[^\n] ");

  /* Read the problem dimensions */
  Vscanf("%ld%ld%*[^\n] ", &n, &kd);
```

```
  /* Read the uplo storage parameter */
  Vscanf("%s%*[^\n] ", nag_enum_arg);
  /* nag_enum_name_to_value(x04nac).
   * Converts NAG enum member name to value
   */
  uplo = nag_enum_name_to_value(nag_enum_arg);
  /* Read the transpose parameter */
  Vscanf("%s%*[^\n] ", nag_enum_arg);
  /* nag_enum_name_to_value(x04nac), see above. */
  trans = nag_enum_name_to_value(nag_enum_arg);
  /* Read the unit-diagonal parameter */
  Vscanf("%s%*[^\n] ", nag_enum_arg);
  /* nag_enum_name_to_value(x04nac), see above. */
  diag = nag_enum_name_to_value(nag_enum_arg);

  /* Read scalar parameters */
  Vscanf(" ( %lf , %lf )%*[^\n] ", &alpha.re, &alpha.im);
  /* Read increment parameter */
  Vscanf("%ld%*[^\n] ", &incx);

  pdab = kd + 1;
  xlen = MAX(1, 1 + (n - 1)*ABS(incx));

  if (n > 0)
    {
      /* Allocate memory */
      if ( !(ab = NAG_ALLOC(pdab*n, Complex)) ||
           !(x = NAG_ALLOC(xlen, Complex)))
        {
          Vprintf("Allocation failure\n");
          exit_status = -1;
          goto END;
        }
    }
  else
    {
      Vprintf("Invalid n\n");
      exit_status = 1;
      return exit_status;
    }

  /* Input matrix AB and vector x*/

  if (uplo == Nag_Upper)
    {
      for (i = 1; i <= n; ++i)
        {
          if (diag == Nag_NonUnitDiag)
            Vscanf(" ( %lf , %lf )", &AB_UPPER(i,i).re,
                   &AB_UPPER(i,i).im);
          for (j = i+1; j <= MIN(i+kd,n); ++j)
            Vscanf(" ( %lf , %lf )", &AB_UPPER(i,j).re,
                   &AB_UPPER(i,j).im);
        }
      Vscanf("%*[^\n] ");
    }
  else
    {
      for (i = 1; i <= n; ++i)
        {
          for (j = MAX(1,i-kd); j < i; ++j)
            Vscanf(" ( %lf , %lf )", &AB_LOWER(i,j).re,
                   &AB_LOWER(i,j).im);
          if (diag == Nag_NonUnitDiag)
            Vscanf(" ( %lf , %lf )", &AB_LOWER(i,i).re,
                   &AB_LOWER(i,i).im);
        }
      Vscanf("%*[^\n] ");
    }
  for (i = 0; i < xlen; ++i)
    Vscanf(" ( %lf , %lf )%*[^\n] ", &x[i].re, &x[i].im);
```

```
  /* nag_ztbsv(f16skc).
   * Solution of complex triangular band system of linear equations.
   *
   */
  nag_ztbsv(order, uplo, trans, diag, n, kd, alpha, ab, pdab, x, incx,
            &fail);
  if (fail.code != NE_NOERROR)
    {
      Vprintf("Error from nag_ztbsv.\n%s\n", fail.message);
      exit_status = 1;
      goto END;
    }

  /* Print output vector x */
  Vprintf("%s\n", " Solution x:");
  for (i = 0; i < xlen; i = ++i)
    {
      Vprintf("( %11f , %11f )\n", x[i].re, x[i].im);
    }

 END:
  if (ab) NAG_FREE(ab);
  if (x) NAG_FREE(x);

  return exit_status;
}
```

9.2 Program Data

```
nag_ztbvs (f16skc) Example Program Data
  4 2                          :Value of n and kd
  Nag_Lower                    :Storage of A
  Nag_NoTrans                  :Transpose A?
  Nag_NonUnitDiag              :Unit diagonal elements?
  ( 1.0, 0.0)                  :Value of alpha
  1                            :Value of incx
 (-1.94, 4.43)
 (-3.39, 3.44) ( 4.12,-4.27)
 ( 1.62, 3.68) (-1.84, 5.53) ( 0.43,-2.66)
               (-2.77,-1.93) ( 1.74,-0.04) ( 0.44, 0.10)  :End of matrix A
 ( -8.86, -3.88)
 (-15.57,-23.41)
 ( -7.63, 22.78)
 (-14.74, -2.40)                                          :End of vector x
```

9.3 Program Results

```
nag_ztbsv (f16skc) Example Program Results

 Solution x:
(    0.000000 ,    2.000000 )
(    1.000000 ,   -3.000000 )
(   -4.000000 ,   -5.000000 )
(    2.000000 ,   -1.000000 )
```